Mode identification from spectroscopy

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Overview

* Why do we need empirical mode identification?

* Modelling of line-profile variations due to NRP

* Spectroscopic mode identification techniques

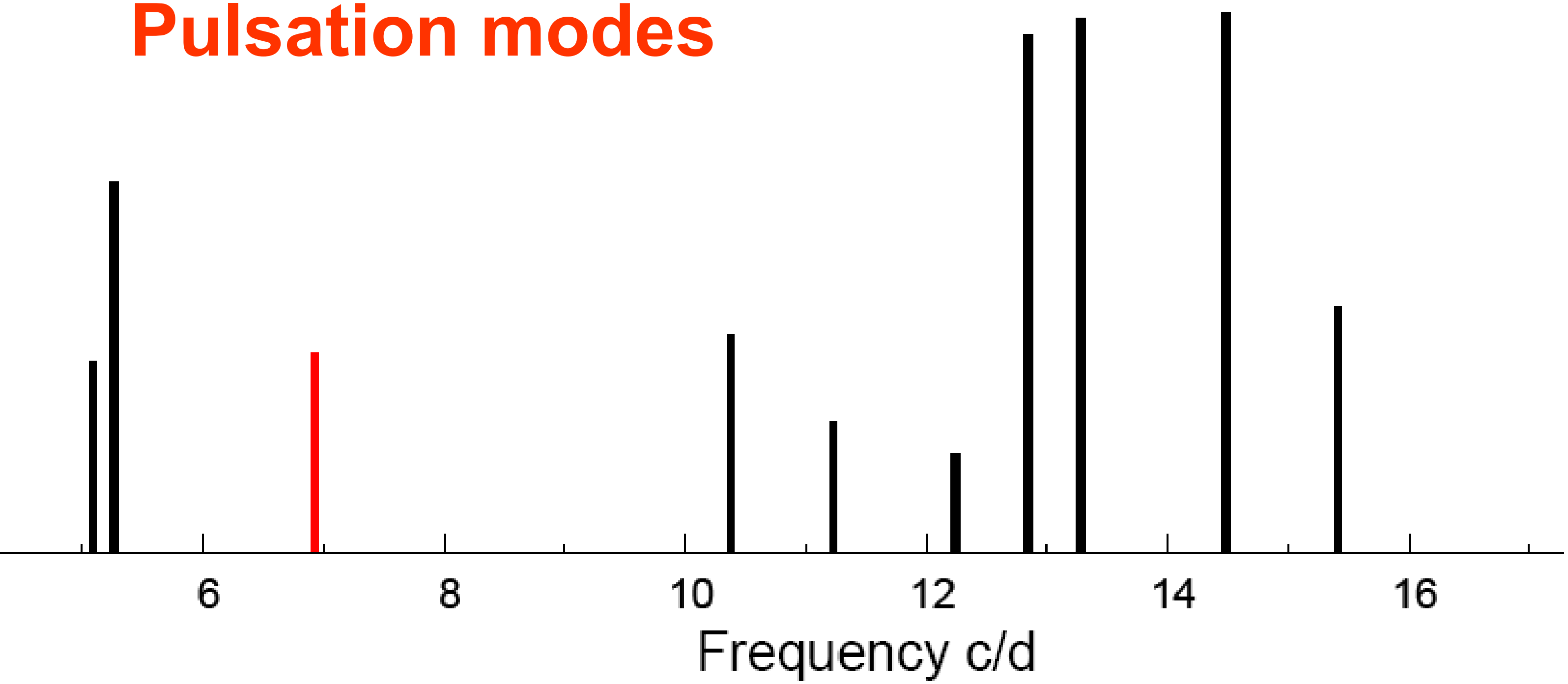
* Generalities

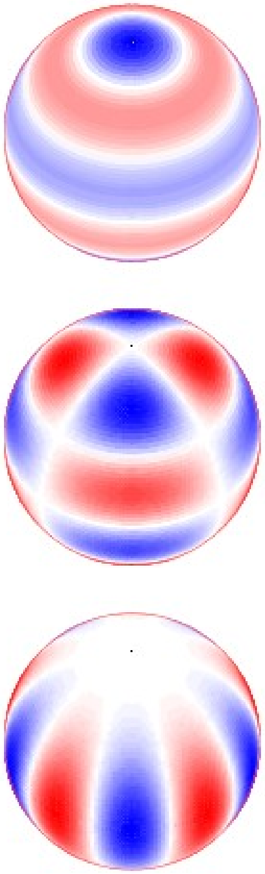
WHY DO WE NEED EMPIRICAL MODE IDENTIFICATION?

ASTEROSEISMOLOGY

To constrain a stellar model,
need of observational pulsational characteristics:

Pulsation frequencies
+
Pulsation modes





Mode identification from spectroscopy

Observational constraints from spectroscopy:

Pulsation frequencies
+
Pulsation modes

Time series of high-resolution high S/N ratio spectroscopic measurements

Basic stellar parameters
+
Chemical abundances

OVERVIEW

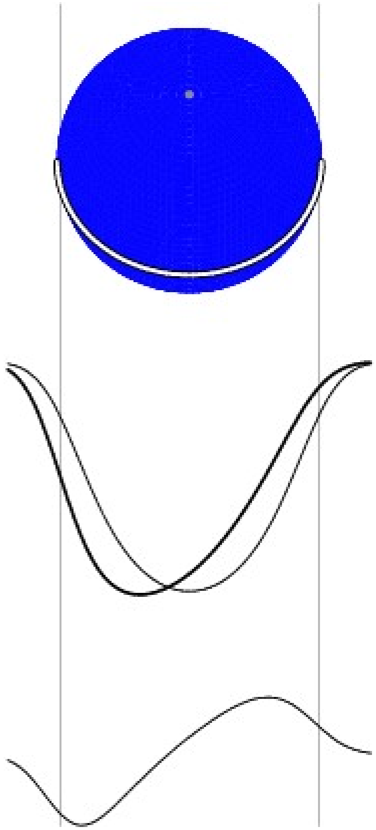
* What causes LPVs?

* Basic line profile model

* Sophisticated line profile model

* Line profile model in FAMIAS

WHAT CAUSES LPVs?



At the stellar surface:

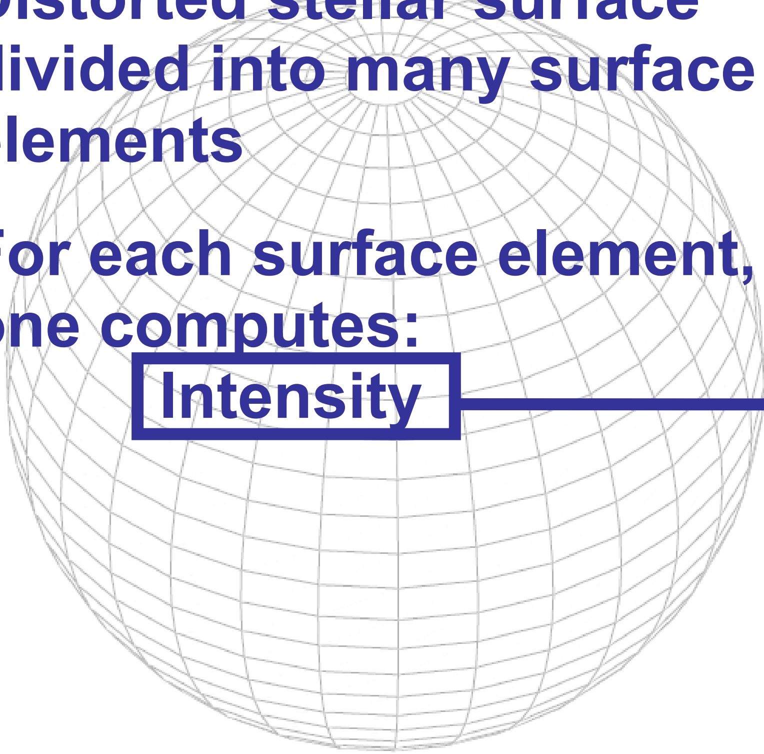
Oscillatory displacements due to pulsation

Periodic temporal variations of

*** velocity field → Doppler shift**

*** local temperature → local brightness, local line profile (width and EW changes)**

BASIC LINE PROFILE MODEL



Distorted stellar surface divided into many surface elements

For each surface element, one computes:

Intensity

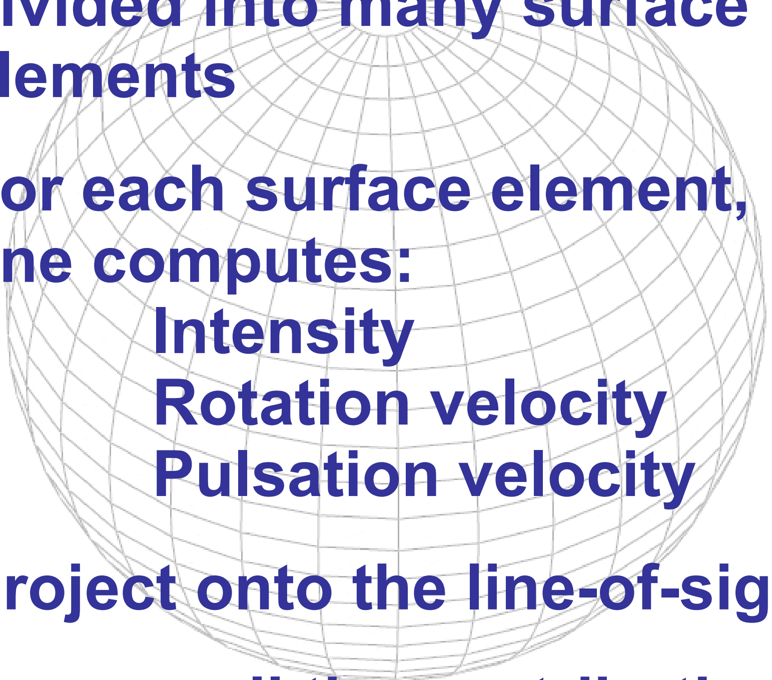
* Weighted by the on the line-of-sight projected area of the surface element

* Doppler shifted by the on the line-of-sight velocity fields caused by rotation and pulsation

Sum up all the contributions of all the visible surface elements

BASIC LINE PROFILE MODEL

Approximations



Distorted stellar surface divided into many surface elements

Spherical stellar surface (not distorted)

(θ, φ)

For each surface element, one computes:

- Intensity
- Rotation velocity
- Pulsation velocity

Project onto the line-of-sight

Sum up all the contributions of all the visible surface elements

BASIC LINE PROFILE MODEL

Distorted stellar surface divided into many surface elements

For each surface element, one computes:

- $\boxed{\text{Intensity}}$
- Rotation velocity
- Pulsation velocity

Project onto the line-of-sight

Sum up all the contributions of all the visible surface elements

Approximations

Gaussian absorption line profile

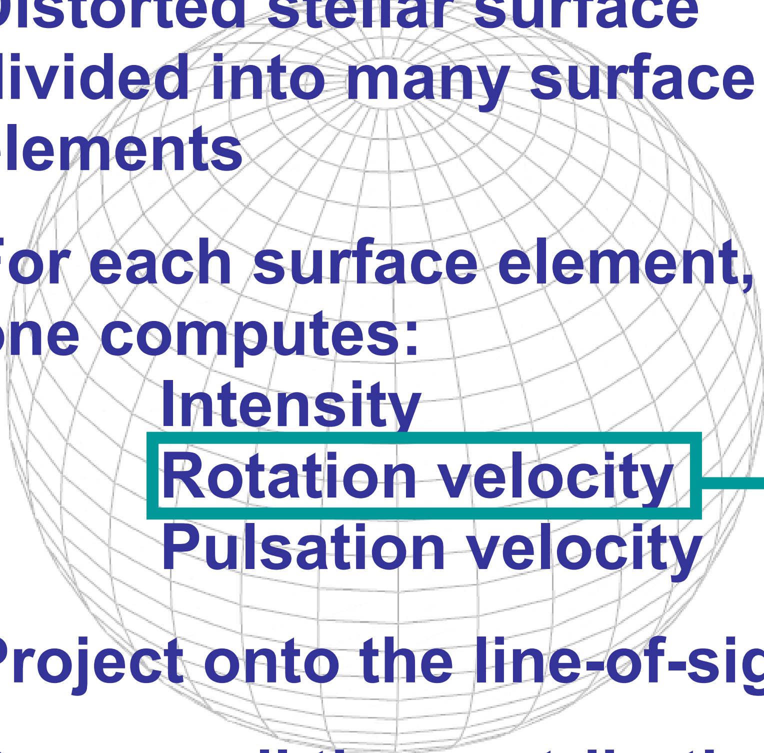
$$1 - \frac{\text{EW}}{\sigma\sqrt{2\pi}}\,\exp\left(-\frac{(\lambda-\lambda_0)^2}{2\sigma^2}\right)$$

Constant in time and over the stellar surface

+ linear limb-darkening law for continuum intensity

$$I_c = I_0\,(1 - u + u\,\cos\chi)$$

BASIC LINE PROFILE MODEL



Distorted stellar surface divided into many surface elements

For each surface element, one computes:

- Intensity
- Rotation velocity
- Pulsation velocity

Project onto the line-of-sight

Sum up all the contributions of all the visible surface elements

Approximations

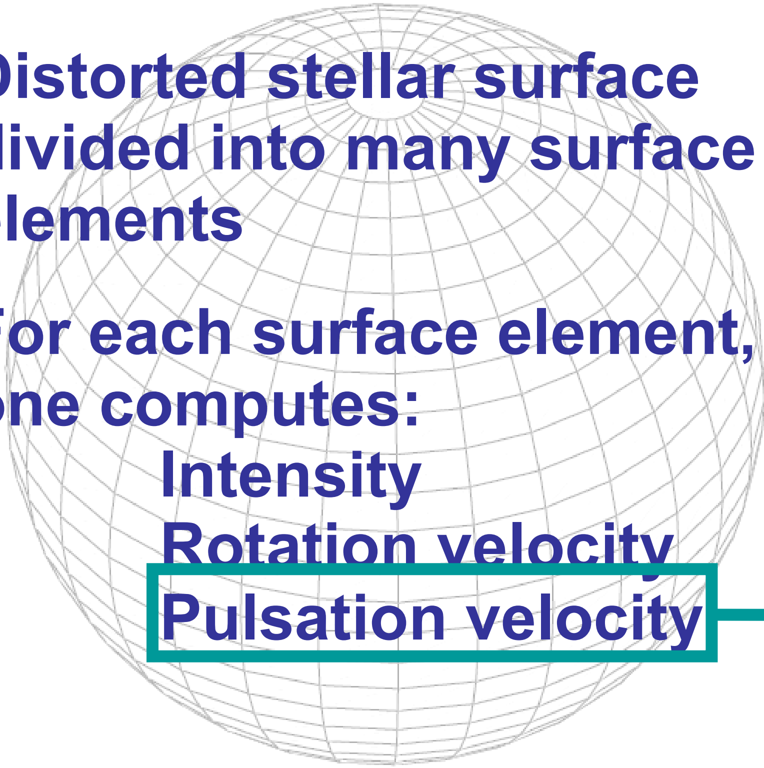
Uniform and time-independent stellar rotation

$$v_{\rm rot}(\theta,\varphi) = v_e \sin i \ \sin\theta \sin\varphi$$

Rotational broadening of spectral line

MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

BASIC LINE PROFILE MODEL



Distorted stellar surface divided into many surface elements

For each surface element, one computes:

- Intensity
- Rotation velocity
- **Pulsation velocity**

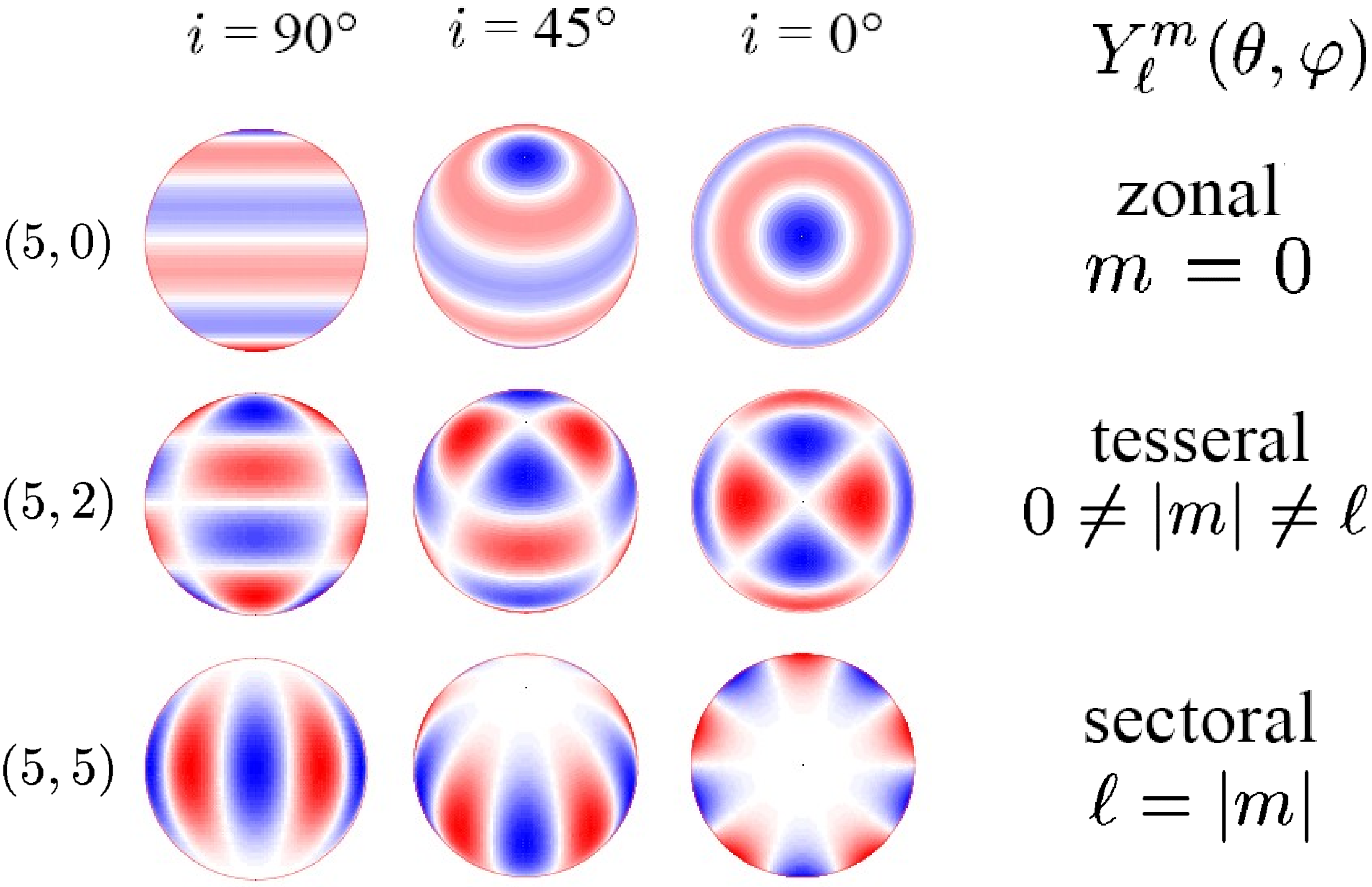
Approximations

- In the linear approximation (i.e. small amplitude of pulsation)
- For a star rotating sufficiently slowly (i.e. neglecting effects of rotation on pulsation)

$$\vec{v}_{\rm puls} = (v_r, v_\theta, v_\varphi) = N_\ell^m v_{\rm p} \left(1, K\frac{\partial}{\partial\theta}, \frac{K}{\sin\theta}\frac{\partial}{\partial\varphi}\right) \boxed{Y_\ell^m(\theta,\varphi)} \exp\left({\rm i}\omega t\right)$$

MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

BASIC LINE PROFILE MODEL



BASIC LINE PROFILE MODEL

Intensity

$$1 - \frac{\boxed{\mathrm{EW}}}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\lambda-\lambda_0)^2}{\boxed{2\sigma^2}}\right)$$

$$I_c = I_0\,(1 - \boxed{u} + u\,\cos\chi)$$

Rotation velocity

$$v_{\mathrm{rot}}(\theta,\varphi) = \boxed{v_e \sin i}\,\sin\theta\sin\varphi$$

Projection onto the line-of-sight $\boxed{i}$

Pulsation velocity

$$N_\ell^m\,\boxed{v_{\mathrm{p}}}\left(1, \boxed{K}\frac{\partial}{\partial\theta}, \frac{K}{\sin\theta}\frac{\partial}{\partial\varphi}\right) Y_\ell^m(\theta,\varphi)$$

$$K = GM/(\omega^2 R^3)$$

* Adopted parameters: EW, *u*, *K*
* Free parameters: (l,m), v_p, *vsini*, *i*, σ

MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

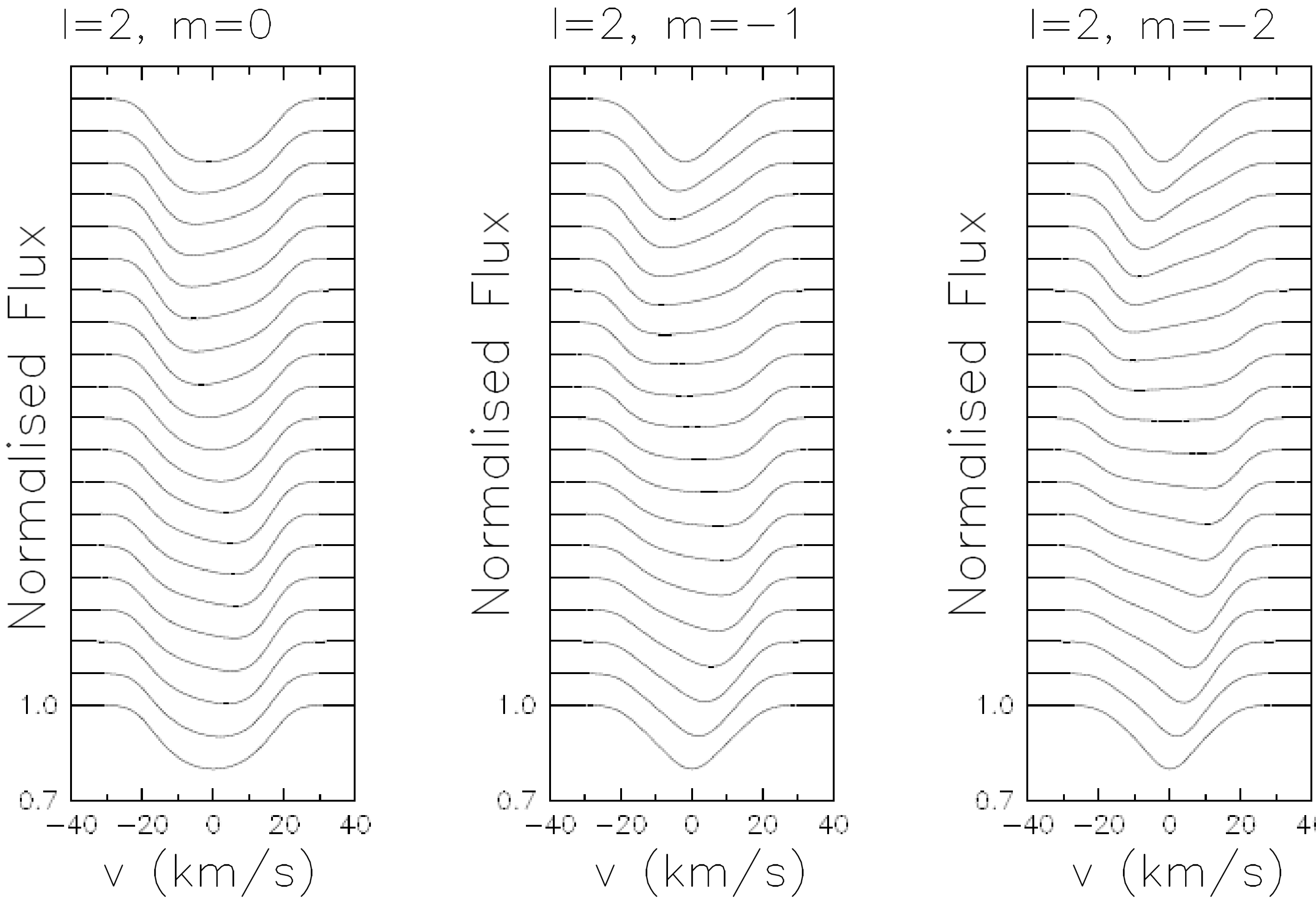
BASIC LINE PROFILE MODEL

$$v_{\rm p} = 5\,\mathrm{km/s}$$

$$v \sin i = 30\,\mathrm{km/s}$$

$$\sigma = 4\,\mathrm{km/s}$$

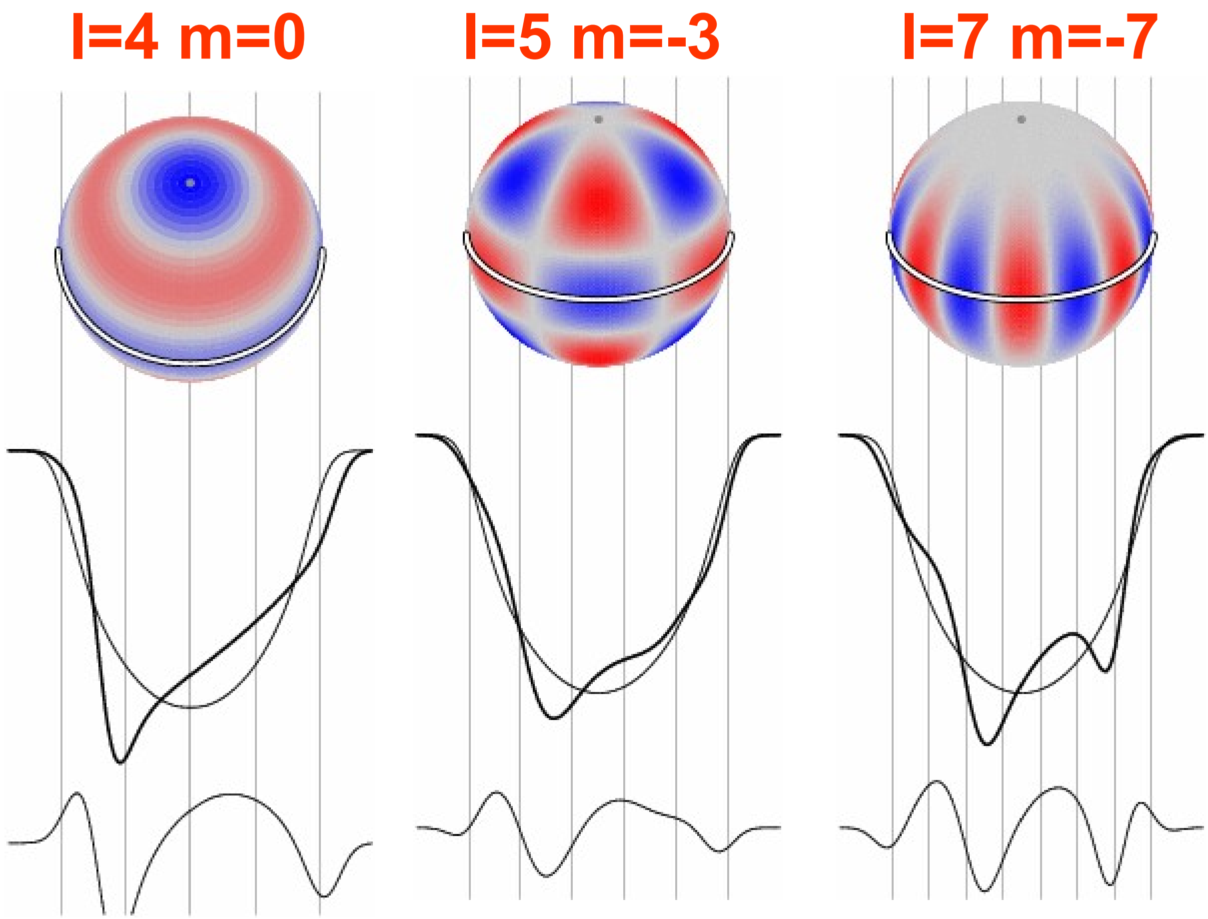
$$i = 55^\circ$$



Spectroscopy allows determination of both l and m while m is not accessible from photometry

MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

BASIC LINE PROFILE MODEL



NRP animation

Creator

by

John Telting

Coen Schrijvers

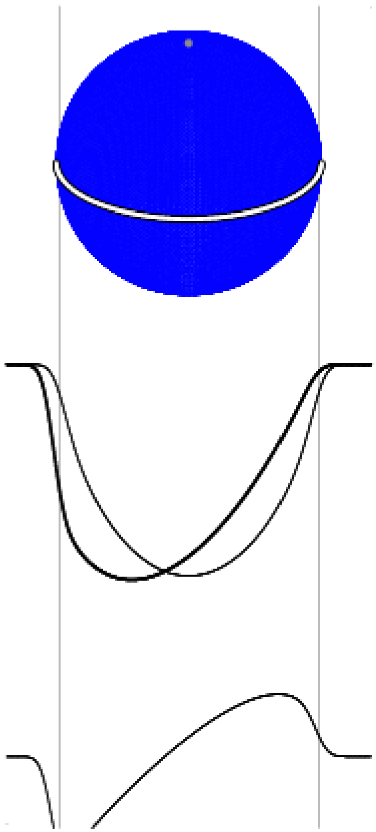
Moving bumps in certain high degree mode

Modes with high degree only visible in spectroscopy

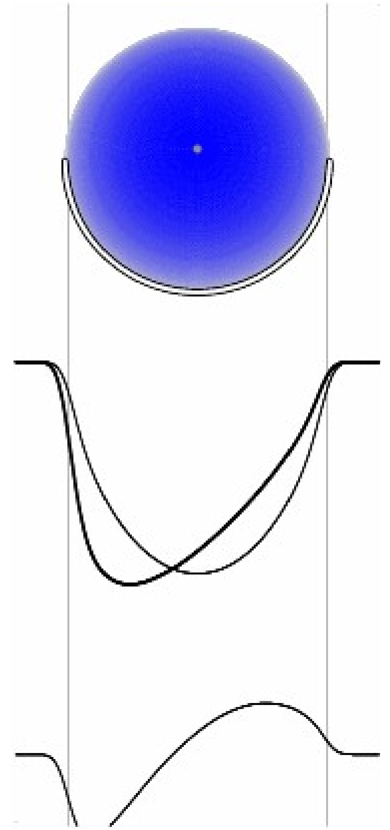
MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

BASIC LINE PROFILE MODEL

(l,m) = (0,0)

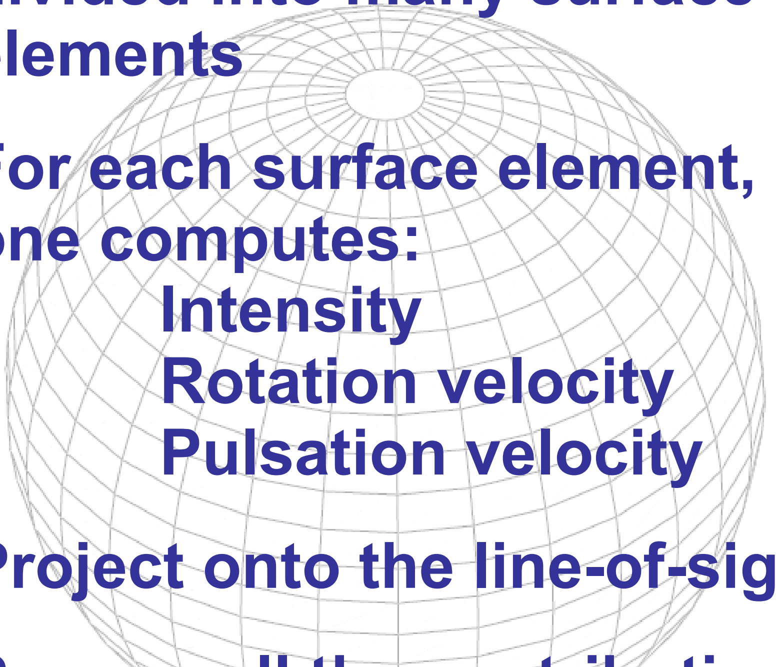


(l,m) = (1,0), i = 0°



Different parameter sets can give the same time series of basic line profile

SOPHISTICATED LINE PROFILE MODEL



Distorted stellar surface divided into many surface elements

Computation of orientation and area of each surface element

For each surface element, one computes:

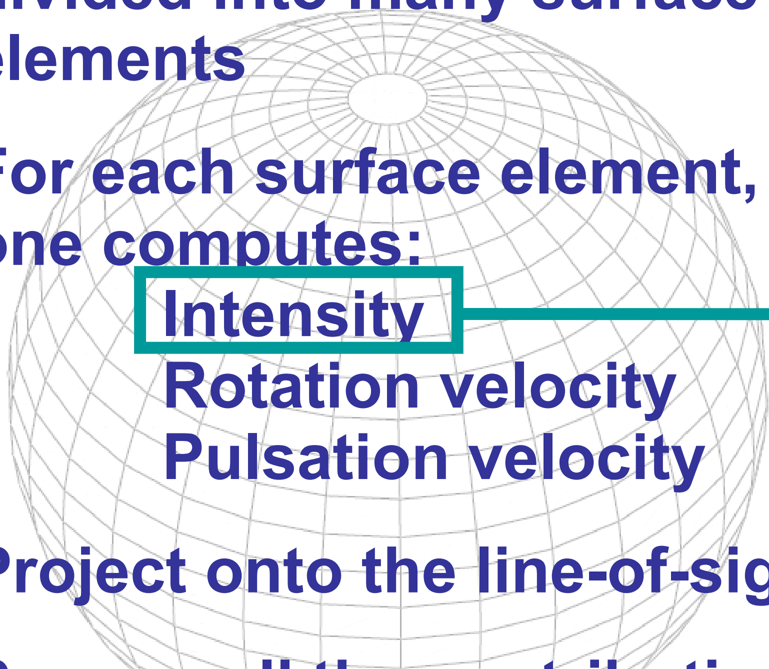
- **Intensity**
- **Rotation velocity**
- **Pulsation velocity**

Project onto the line-of-sight

Sum up all the contributions of all the visible surface elements

MODELLING OF LINE-PROFILE VARIATIONS DUE TO NRP

SOPHISTICATED LINE PROFILE MODEL



Distorted stellar surface divided into many surface elements

For each surface element, one computes:

$\boxed{\textbf{Intensity}}$
Rotation velocity
Pulsation velocity

Project onto the line-of-sight

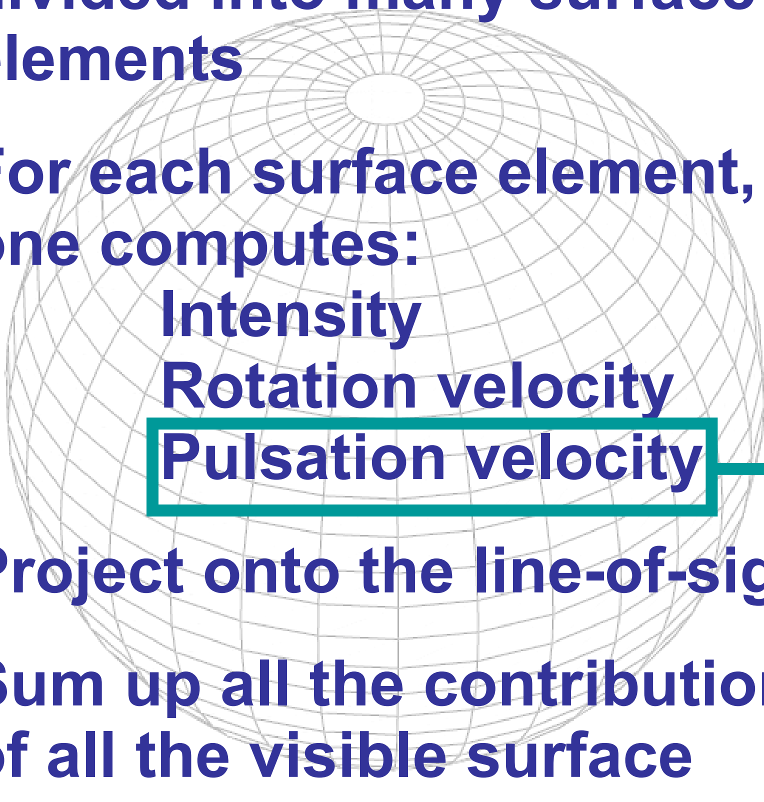
Sum up all the contributions of all the visible surface elements

Pre-computed intensity spectra calculated for a given T_{eff} and log g and cos λ with a stellar atmosphere code

Local T_{eff} and log g vary in time

- Intensity varies in time
- Local line profile varies in time, i.e. both time varying width and EW

SOPHISTICATED LINE PROFILE MODEL



Distorted stellar surface divided into many surface elements

For each surface element, one computes:

- **Intensity**
- **Rotation velocity**
- **Pulsation velocity**

Project onto the line-of-sight

Sum up all the contributions of all the visible surface elements

Improved formalism that takes into account the influence of the rotation on pulsation

Inclusion of Coriolis correction terms

LINE PROFILE MODEL IN FAMIAS

Intensity

- Gaussian intrinsic line profile
- Quadratic limb-darkening law
- Brightness variations
- Parameterized variable equivalent width due to temperature variations

Rotation velocity

- Uniform and time-independent
- Slow

Pulsation velocity

- Linear pulsation
- Effects of the Coriolis force to the first order taken into account

Zima (2006)

OVERVIEW

* Observations required

* The methods

OBSERVATIONS REQUIRED

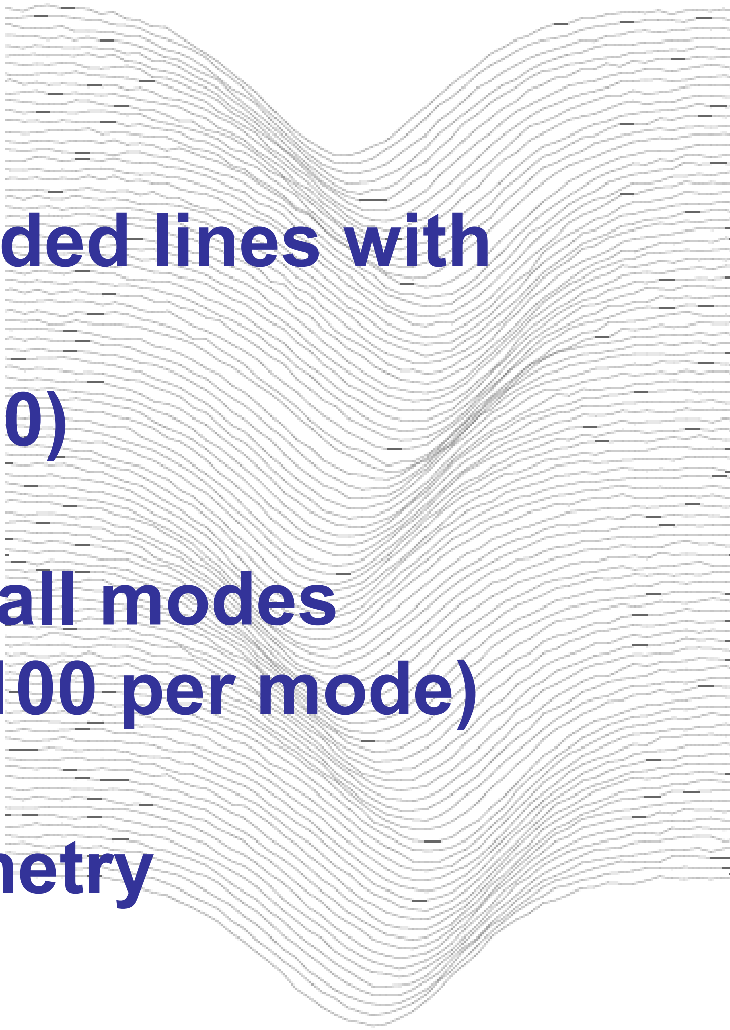
Ideally, use of isolated non-blended lines with

- **High S/N ratio (> 200)**
- **High resolution (R > 40000)**

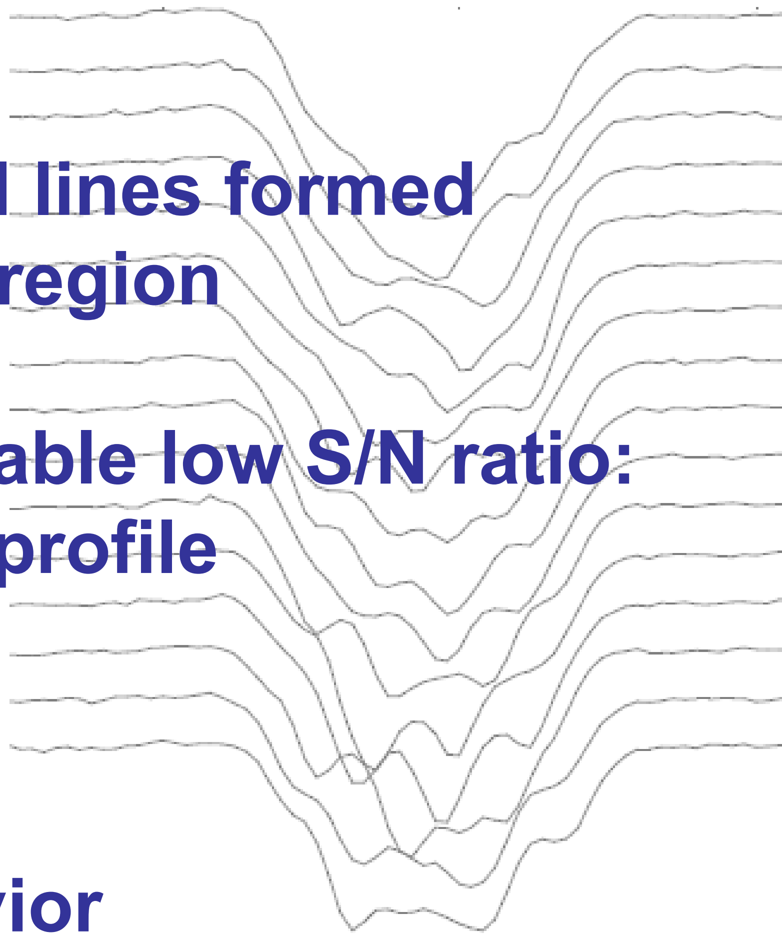
Ideally, covering entire cycle of all modes

 # > several hundred (say 100 per mode)

Ideally, accompanied by photometry



OBSERVATIONS REQUIRED



To increase S/N ratio:

Use of average of several lines formed in the same line-forming region

For too faint star with unavoidable low S/N ratio:

Use of cross-correlation profile

Assumption:

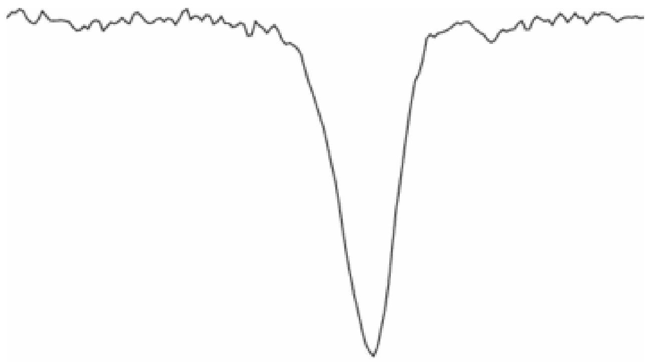
all used lines show the same temporal behavior

Silicon lines for pulsating B-type stars

- Sufficiently strong without being much affected by blending
- Dominated by thermal broadening → Gaussian profile
- LPVs little affected by temperature variations at the stellar surface

Si II lines for SPBs
Si III lines for β Cephei stars

LPVs for the Si III 4552 Å line of the β Cephei star 12 Lac

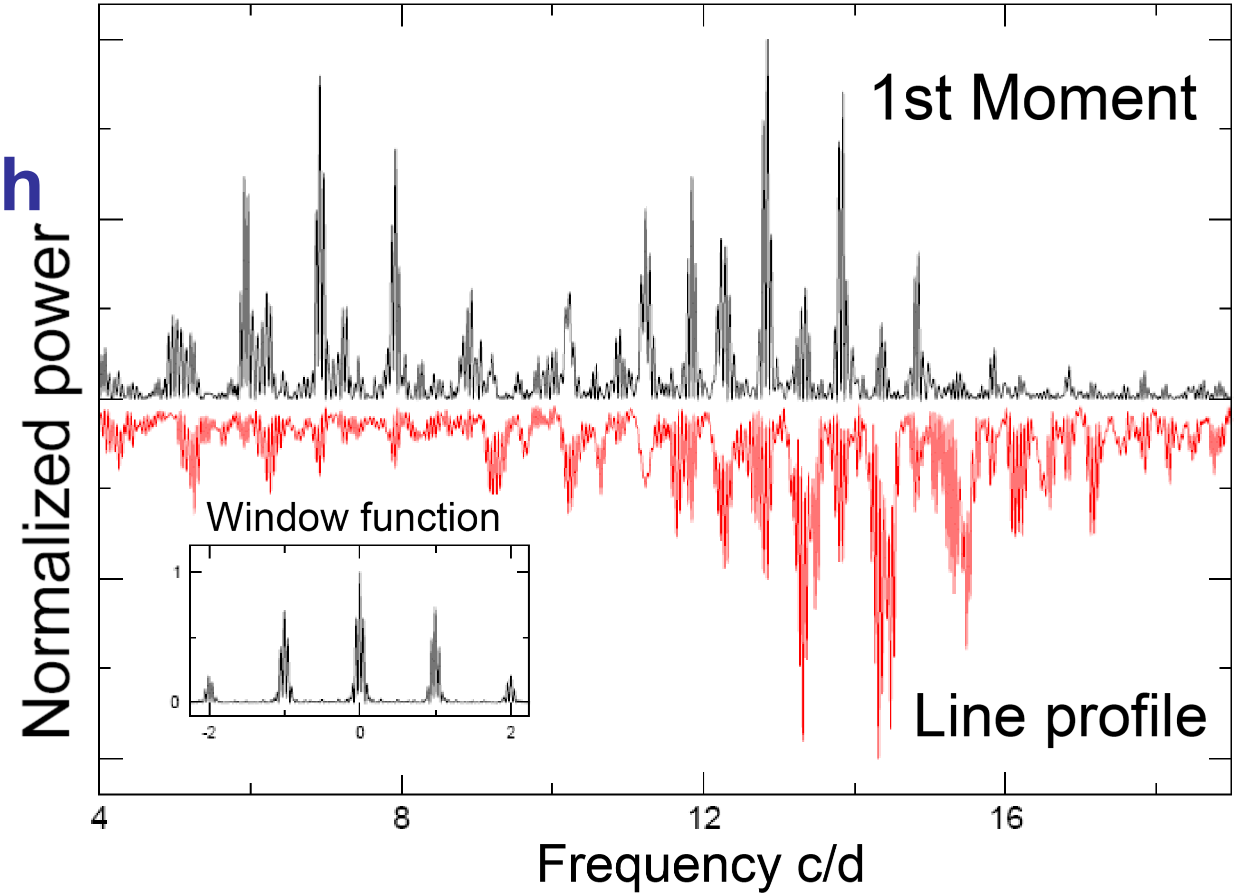


OBSERVATIONS REQUIRED

Pulsation frequencies unambiguously determined

Frequency analysis of both **radial velocity** and **pixel-by-pixel** variations

+ photometry



THE METHODS

* Line-profile fitting technique
* The moment method
* The IPS and pixel-by-pixel method

THE METHODS - Line-profile fitting technique

Theoretically computed LPVs for different values of (l,m) and for the other parameters

Observed line profile variations

Goodness of fit measure e.g. based on least squares

Set of "best fitting parameters"

Ledoux (1951), Osaki (1971), Smith (1977), etc.

THE METHODS - Line-profile fitting technique

- Not only (l,m) is determined but also the other parameters, such as the amplitude of the mode, the inclination angle and the rotational equatorial velocity

BUT

- Extremely CPU-time consuming
    * If no thorough investigation of parameter space, not sure to find the best fitting models
    * Simultaneous identification of multiple modes is unrealistic

- Depends much on the theoretical model

THE METHODS - Line-profile fitting technique

To decrease computational time:

Use of line profiles folded in only several bins for each detected frequency, such that the variations of other modes are assumed to cancel out

BUT

Phase binning is equivalent to extending the exposure times of the spectra

↓

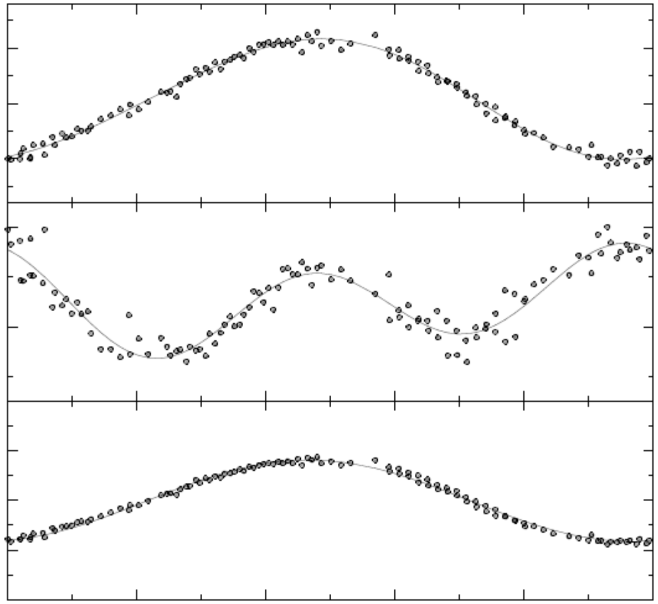
Phase smearing which can have an impact on the mode identification results

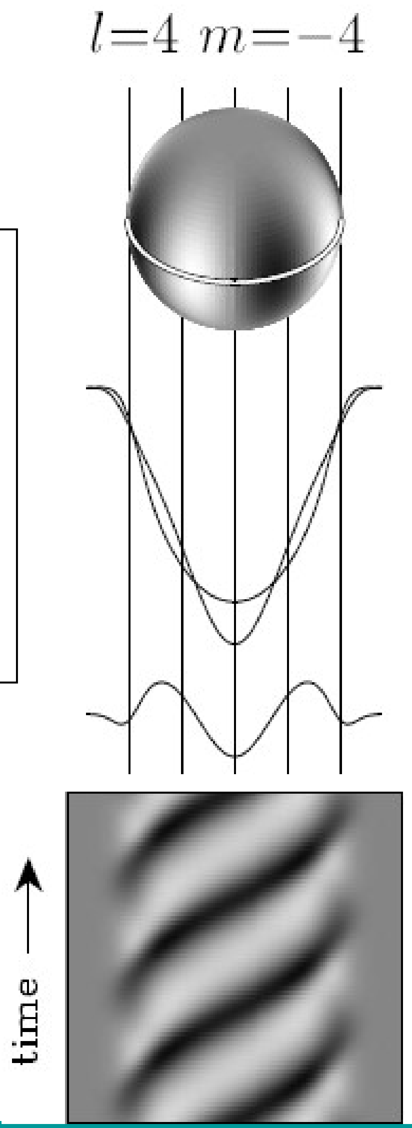
THE METHODS

The entire absorption line profile can be replaced by

First few moments of a line profile, which are integrated quantities over the profile

Intensity information of each wavelength bin across the line profile





THE METHODS

Theoretically computed
* moments
* bin intensities

Observed
* moments
* bin intensities

* Moment method

* IPS
  Pixel-by-pixel method

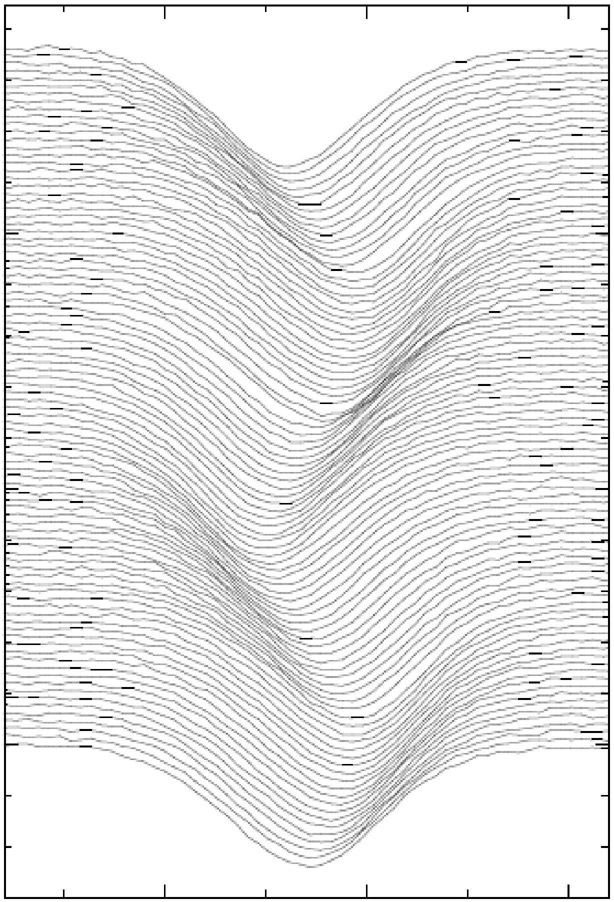
Goodness of fit measure

Set of “best fitting parameters”
In particular, identification of (l,m)

THE METHODS - The moment method

Integration over the whole line profile p(v,t)

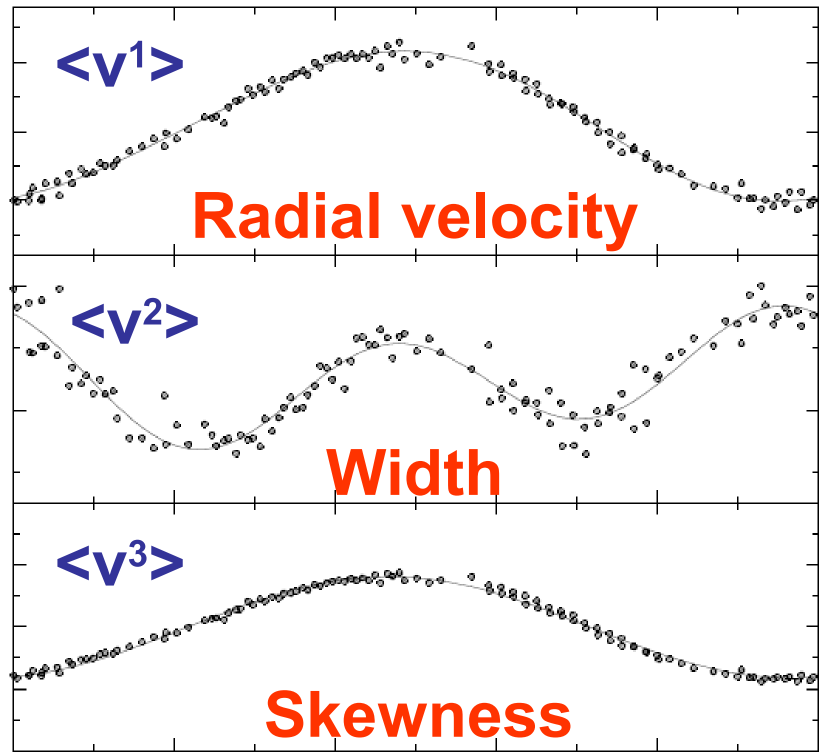
The first three velocity moments



$$M_n(t) \equiv \int_{-\infty}^{+\infty} dv\, v^n\, (1 - p(v,t))$$

$$\langle v^n \rangle \equiv \frac{M_n}{M_0}$$

$\equiv$ EW



Aerts et al. (1992)

THE METHODS - The moment method

- Less CPU-time consuming
  * Thorough investigation of parameter space possible
  * Simultaneous identification of multiple modes feasible for a few modes (without using phase binning)

- Less model dependent
  * Not very sensitive to EW variations
  * Only assumption on the local line profile: it is symmetric, e.g. local line profile approximated with a constant Voigt function

BUT

Use of integrated quantities $\rightarrow$ only for low degree mode ($l \leq 4$)

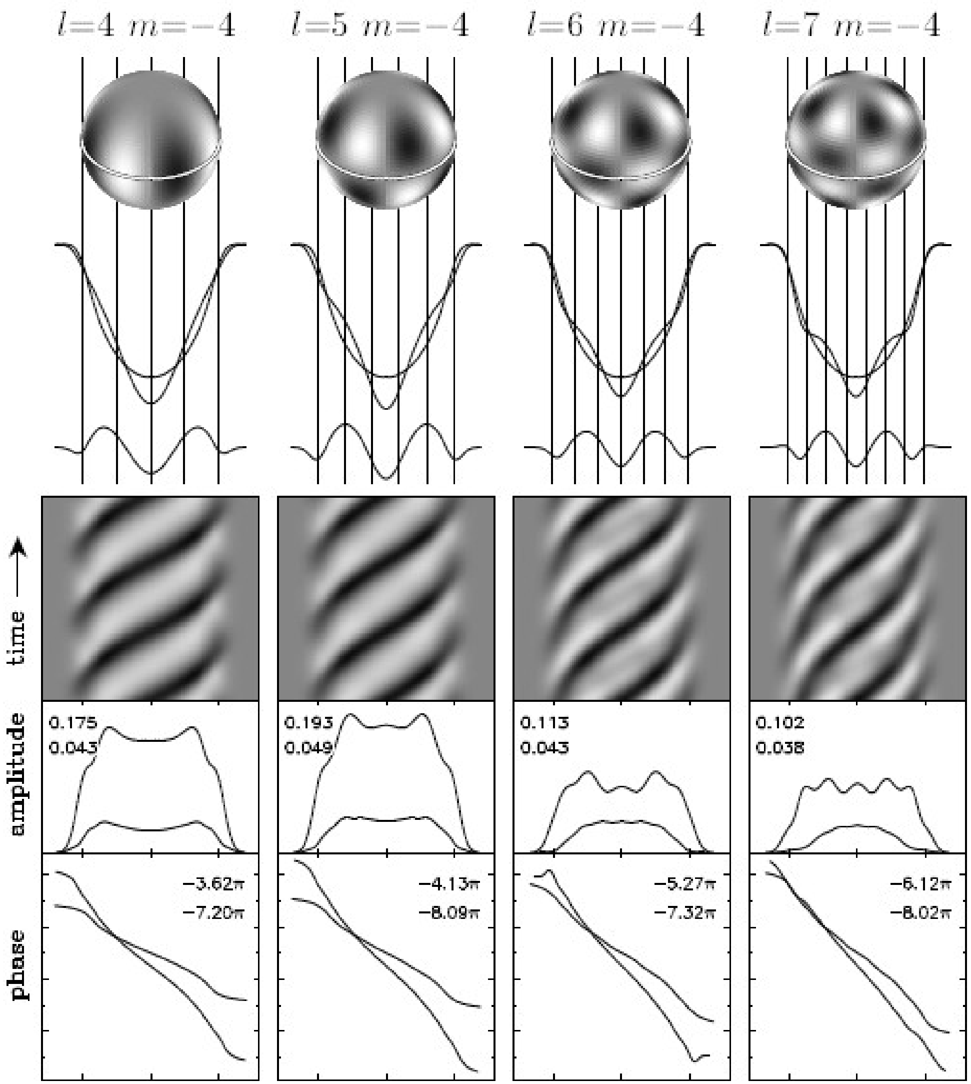
THE METHODS - IPS and pixel-by-pixel method

For every wavelength bin,
for each detected pulsation frequency,
computation of zero point, amplitude and phase
using a multi-periodic least-squares fit
with fitting formula as follows

$$
\begin{aligned}
p(v,t) = C(v) &+ A_0(v)\ \sin(\sigma t + \boxed{\Psi_0(v))} \longleftrightarrow \ell \\
&+ A_1(v)\ \sin(2\sigma t + \boxed{\Psi_1(v))} \longleftrightarrow m \\
&+ \cancel{A_2(v)\ \sin(3\sigma t} + \Psi_2(v))
\end{aligned}
$$

Determining (l,m) from phase distributions across the line profile

THE METHODS - Intensity period search (IPS)



Extensive numerical simulations by Telting & Schrijvers (1997):

$$\ell \approx 0.10 + 1.09\,|\Delta\Psi_0|/\pi,$$

$$|m| \approx -1.33 + 0.54\,|\Delta\Psi_1|/\pi$$

Where maximum red-to-blue phase difference

of detected frequency f: $\Delta\Psi_0$

of first harmonic of f: $\Delta\Psi_1$

THE METHODS – Intensity period search (IPS)

Phase diagrams contain mostly information about l and |m|
→ direct identification without having to model the pulsation

BUT

- Amplitude of the first harmonic of a frequency may be very low → need of very high S/N ratio (> 300)
- Method fails for stars with low vsini
- No information about the other parameters
- Uncertainty on l and m relatively large for low-degree modes
  Error for l: ± 1
  Error for m: ± 2

THE METHODS - The pixel-by-pixel method

Mantegazza (2000)

For every wavelength bin,
for each detected pulsation frequency,
computation of Z, A_i and ϕ_i

$$Z + \sum_i A_i \sin(2\pi(f_i t + \phi_i))$$

For each detected pulsation frequency,
use of the zero point, amplitude and phase
to compute 10 profiles evenly distributed across one pulsation cycle

Direct line-profile fitting to this mono-mode profile

THE METHODS - The pixel-by-pixel method

Mantegazza (2000)

Allows identification of multiple modes without limits for (l,m)

BUT

- **Very small value of vsini can prevent mode identification**
- **Method fails for stars whose dominant mode has high-amplitude relative to the projected rotational velocity**
- **No statistical significance limit of the derived identifications**

→ **Fourier Parameter Fit method by Zima (2006)**

GENERALITIES

- Methods successfully applied to δ Scuti stars and β Cephei stars, applicable to all main-sequence pulsators hotter than the Sun

- The azimuthal order m and its sign can be determined by both the moment method and the pixel-by-pixel method

  In FAMIAS, a positive value of m denotes a prograde-mode, i.e. propagating in the direction of the stellar rotation

BUT

The degree l is usually not determined unambiguously

GENERALITIES

- Apply both the pixel-by-pixel method (FPF method by Zima 2006) and the moment method (Briquet & Aerts 2003)

- The moment method is better suited than the FPF method
- *** when *vsini* has a very small value (*vsini* < 10 km/s)**
- *** when the pulsation velocity is large relative to the projected rotational velocity**

- The FPF method is better suited than the moment method for high-degree modes (l > 4)

- If both photometry and spectroscopy available:
- *** search for frequencies in both of them**
- *** use photometric mode identification for l and fix this in spectroscopic mode identification**

Mode identification from spectroscopy

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